MathType

Statistics Examples

Mean, Variance, and Standard Deviation

Let $X_1, X_2, \ldots, X_n$ be *n* observations of a random variable *X*. We wish to measure the average of $X_1, X_2, \ldots, X_n$ in some sense. One of the most commonly used statistics is the mean, μ_X, defined by the formula

$$\mu_X = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Next, we wish to obtain some measure of the variability of the data. The statistics most often used are the variance σ_X^2 and the standard deviation $\sigma_X = \sqrt{\sigma_X^2}$. We have

$$\sigma_X = \sqrt{\frac{1}{n}\left\{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2\right\}}$$

It is easy to show that the variance is simply the mean squared deviation from the mean.

Covariance and Correlation

Next, let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be *n* pairs of values of two random variables *X* and *Y*. We wish to measure the degree to which *X* and *Y* vary together, as opposed to being independent. The first statistic we will calculate is the covariance σ_{XY} given by

$$\sigma_{XY} = \frac{1}{n}\left\{\sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)\right\}$$

Actually, a much better measure of correlation can be obtained from the formula

$$\rho_{XY} = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{\sqrt{\left\{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2\right\}\left\{\sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2\right\}}}$$

The quantity ρ_{XY} is known as the coefficient of correlation of *X* and *Y*.

Design Science, Inc.
www.mathtype.com

4028 Broadway Long Beach California 90803 USA | Phone 562.433.0685 | Fax 562.433.6969 | Email info@mathtype.com

The Covariance Matrix

Covariances and variances are sometimes arranged in a matrix known as a covariance matrix. In our case, the covariance matrix will be a 2×2 matrix:

$$\mathbf{C}=\begin{bmatrix}\sigma_X^2 & \sigma_{XY}\\ \sigma_{XY} & \sigma_Y^2\end{bmatrix}$$

The eigenvalues of the covariance matrix are sometimes of interest. These are obtained in the usual way by solving the characteristic equation:

$$p(\lambda)=\det(\lambda\mathbf{I}-\mathbf{C})=\begin{vmatrix}\lambda-\sigma_X^2 & -\sigma_{XY}\\ -\sigma_{XY} & \lambda-\sigma_Y^2\end{vmatrix}$$